applied to the last result, give after simplifications,

$$\{(M+m)R^2+Mk'^2\}\ddot{\varphi}-mRr'\cos(\theta-\alpha)\ddot{\theta}+mRr'\sin(\theta-\alpha)\dot{\theta}^2=g(M+m)R\sin\alpha,$$

$$Rr'\cos(\theta-\alpha)\ddot{\varphi}-(r'^2+k''^2)\ddot{\theta}=gr'\sin\theta.$$

Eliminating $\ddot{\varphi}$,

$$[(r'^2+k''^2)\{(M+m)R^2+Mk'^2\}+mR^2r'^2\cos^2(\theta-\alpha)]\ddot{\theta}-mR^2r'^2\sin(\theta-\alpha)\cos(\theta-\alpha)\dot{\theta}^2$$
$$=-gr'[\{(M+m)R^2+Mk'^2\}\sin\theta+(M+m)R^2\sin\alpha\cos(\theta-\alpha)].$$

Multiplying by $2\dot{\theta}$ and integrating

$$[(r'^2+k''^2)\{(M+m)R^2+Mk'^2\}+mr'^2R^2\cos^2(\theta-\alpha)]\dot{\theta}^2$$
$$=gr'[2\{(M+m)R^2+Mk'^2\}\cos\theta-2(M+m)R^2\sin\alpha\sin(\theta-\alpha)]+C',$$

which is of the same general form as (7), p. 351, this MONTHLY for November, 1916.

NUMBER THEORY.

235. Proposed by W. D. CAIRNS, Oberlin College.

Prove that $n=1$ is the only positive integer for which n^4+4 is a prime.

SOLUTION BY WM. E. PATTEN, Government Institute of Technology, Shanghai, China.

$$n^4+4=(n^4+4n^2+4)-4n^2=(n^2+2)^2-(2n)^2=(n^2+2n+2)(n^2-2n+2).$$

Therefore, n^4+4 is a prime, if at all, only for those values of n which make either $n^2+2n+2=1$, or $n^2-2n+2=1$, since each of the factors of n^4+4 given above is integral in value when n is integral, and both are positive when n is positive.

(1) When $n^2+2n+2=1$, then $n=-1$.

(2) When $n^2-2n+2=1$, then $n=+1$. When $n=+1$, then $n^4+4=5$, a prime.

Therefore, n^4+4 is a prime for $n=1$, and for no other positive integral values of n.

Also solved by ELMER SCHUYLER, FRANK IRWIN, HORACE OLSON, ELIJAH SWIFT, H. H. CLARK, ELIZABETH B. DAVIS, NORMAN ANNING, L. G. WELD, and the PROPOSER.

236. Proposed by V. M. SPUNAR, Chicago, Illinois.

Find integral values of x, y, z, such that

$$xy+z=\square,\quad yz+x=\square,\quad\text{and}\quad xz+y=\square.$$

SOLUTION BY ARTEMAS MARTIN, LL.D., Washington, D. C.

Assume $x=n^2$, $y=(n+1)^2$; then the given equation becomes

$$n^2(n+1)^2+z=\square,\quad(n+1)^2z+n^2=\square,\quad n^2z+(n+1)^2=\square.$$

Put

$$n^2(n+1)^2+z=a^2.$$

Assume $a=n^2+n+b$, and the last equation becomes

$$n^2(n+1)^2+z=a^2=(n^2+n+b)^2,$$

from which we immediately find

$$z=b(2n^2+2n+b).$$

Substituting in

$$(n+1)^2z+n^2=\square,$$

we have

$$b(n+1)^2(2n^2+2n+b)+n^2=\square=c^2,$$

which is satisfied by $b = 2$; for then

$$c^2 = 4n^4 + 12n^3 + 17n^2 + 12n + 4 = (2n^2 + 3n + 2)^2,$$

and $z = 4(n^2 + n + 1)$. This value of z, with the assumed values, $x = n^2$, $y = (n + 1)^2$, satisfies all the proposed conditions.

$$xy + z = n^2(n + 1)^2 + 4(n^2 + n + 1) = (n^2 + n + 2)^2,$$

$$yz + x = 4(n + 1)^2(n^2 + n + 1) + n^2 = (2n^2 + 3n + 2)^2,$$

$$xz + y = 4n^2(n^2 + n + 1) + (n + 1)^2 = (2n^2 + n + 1)^2.$$

If $n = 1$, then $x = 1$, $y = 4$, $z = 12$.
If $n = 2$, then $x = 4$, $y = 9$, $z = 28$.
If $n = 3$, then $x = 9$, $y = 16$, $z = 52$.
And so on, indefinitely.

The values of x, y, z just found will also satisfy the conditions

$$xy + x + y = \square, \quad xz + x + z = \square, \quad \text{and} \quad yz + y + z = \square.$$

Also solved by ELIZABETH B. DAVIS and H. N. CARLETON.

237. Proposed by NORMAN ANNING, Chilliwack, B. C.

Prove that for three numbers x, y, z,

$$9\Sigma(x - y)^4 = \Sigma(2x - y - z) = 2\square.$$

SOLUTION BY E. F. CANADAY, University of South Dakota.

This problem is evidently misprinted. If we write it

$$9\Sigma(x - y)^4 = \Sigma(2x - y - z)^4 = 2\square,$$

a solution is possible. To prove

$$9[(x - y)^4 + (y - z)^4 + (z - x)^4] = (2x - y - z)^4 + (2y - z - x)^4 + (2z - x - y)^4 = 2\square,$$

we put

$$(x - y) = a, \quad (y - z) = b, \quad \text{and} \quad (z - x) = -(a + b).$$

Then

$$9[a^4 + b^4 + (- a - b)^4] = (2a + b)^4 + (b - a)^4 + (- a - 2b)^4 = 9(2a^4 + 4a^3b + 6a^2b^2 - 4ab^3 + 2b^4) = 16a^4 + 32a^3b + 24a^2b^2 + 8ab^3 + b^4 + b^4 - 4ab^3 + 6a^2b^2 - 4a^3b + a^4 + a^4 + 8a^3b + 24a^2b^2 + 32ab^3 + 16b^4 = 2[9(a^4 + 2a^3b + 3a^2b^2 + 2ab^3 + b^4)] = 18a^4 + 36a^3b + 54a^2b^2 + 36ab^3 + 18b^4 = 2\square.$$

$$2[3(a^2 + ab + b^2)]^2 = 2\square.$$

Also solved by the PROPOSER.

QUESTIONS AND DISCUSSIONS.

SEND ALL COMMUNICATIONS TO U. G. MITCHELL, University of Kansas, Lawrence, Kansas.

REPLIES.

20. Some of our readers would like to have a simple account, without proofs, of just what has been accomplished toward the proof of the theorem that the equation $x^n + y^n = z^n$ is impossible in integers when $n > 2$.

Readers interested in the above question will be glad to learn that a better and more complete article than that contemplated as an answer to the question